

Code Name:

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## Fall 2009 Math 245 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use your book and/or notes, but no calculators or other aids. This exam will last 50 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Total:	50		100

Problem 1. Carefully define the following terms:

a. constructive proof

b. cardinal number

c.  $\Delta$

d. power set

e. (set) union

Problem 2. Let  $U = \{a, b, c, d\}$ ,  $A = \{a, a, b, c\}$ ,  $B = \{a, c\}$ ,  $C = \{a, c, d\}$ .  
Find  $((A - B) \Delta C) \cap (C \cap B)^c$ .

Problem 3. (9.5 in text) Prove that if  $n$  is an even integer then  $\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$ .

Problem 4. (16.4 in text) Let  $A, B$  be two sets. Prove that if  $A \subseteq B$  then  $B^c \subseteq A^c$ .

Problem 5. Prove that, for all  $n \in \mathbb{N}$ ,  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ .

Problem 6. Use the Euclidean algorithm to calculate  $\gcd(605, 847)$ .

Problem 7. For all odd integers  $n$ , prove that  $n^3$  is odd.

Problem 8. Prove that  $\sqrt{3}$  is irrational.

Problem 9. Prove that  $x^2 + 2x < 8$  if and only if  $|x + 1| < 3$ .

Problem 10. Consider the two-element Boolean algebra  $\{0, 1\}$ . Prove the absorption theorem:  $\forall a \forall b, a \oplus (a \odot b) \equiv a$ .